1991 - AB1

1. Let $f$ be the function that is defined for all real numbers $x$ and that has the following properties.

(i) $f''(x) = 24x - 18$ (ii) $f'(1) = -6$ (iii) $f(2) = 0$

(a) Find each $x$ such that the line tangent to the graph of $f$ at $(x, f(x))$ is horizontal.

(b) Write an expression for $f(x)$.

(c) Find the average value of $f$ on the interval $1 \le x \le 3$.

1991 - AB2

2. Let $R$ be the region between the graphs of $y = 1 + \sin(\pi x)$ and $y = x^2$ from $x = 0$ to $x = 1$.

(a) Find the area of $R$.

(b) Set up, but <u>do not integrate</u> an integral expression in terms of a single variable for the volume of the solid generated when $R$ is revolved about the <u>$x$-axis</u>.

(c) Set up, but <u>do not integrate</u> an integral expression in terms of a single variable for the volume of the solid generated when $R$ is revolved about the <u>$y$-axis</u>.

1991 - AB3

3. Let $f$ be the function defined by $f(x) = (1 + \tan x)^{\frac{3}{2}}$ for $-\frac{\pi}{4} < x < \frac{\pi}{2}$.

(a) Write an equation for the line tangent to the graph of $f$ at the point where $x = 0$.

(b) Using the equation found in part (a), approximate $f(0.02)$.

(c) Let $f^{-1}$ denote the inverse function of $f$. Write an expression that gives $f^{-1}(x)$ for all $x$ in the domain of $f^{-1}$.

1991 - AB4

4. Let $f$ be the function given by $f(x) = \dfrac{|x| - 2}{x - 2}$.

(a) Find all the zeros of $f$.

(b) Find $f'(1)$.

(c) Find $f'(-1)$.

(d) Find the range of $f$.

1991 - AB 5

Let $f$ be a function that is even and continuous on the closed interval $[-3, 3]$. The function $f$ and its derivatives have the properties indicated in the table below.

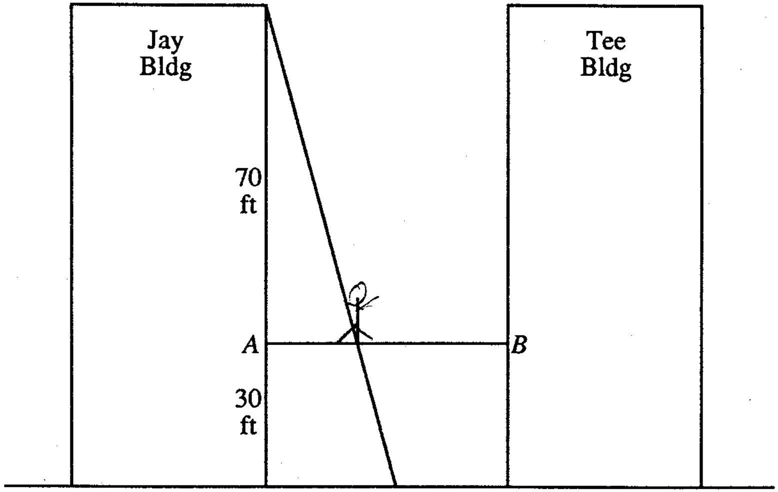
| $x$ | 0 | $0 < x < 1$ | 1 | $1 < x < 2$ | 2 | $2 < x < 3$ |
|---|---|---|---|---|---|---|
| $f(x)$ | 1 | Positive | 0 | Negative | $-1$ | Negative |
| $f'(x)$ | Undefined | Negative | 0 | Negative | Undefined | Positive |
| $f''(x)$ | Undefined | Positive | 0 | Negative | Undefined | Negative |

(a) Find the $x$-coordinate of each point at which $f$ attains an absolute maximum value or an absolute minimum value. For each $x$-coordinate you give, state whether $f$ attains an absolute maximum or an absolute minimum.

(b) Find the $x$-coordinate of each point of inflection on the graph of $f$. Justify your answer.

(c) In the $xy$-plane provided below, sketch the graph of a function with all the given characteristics of $f$.

Note: The $xy$-plane is provided in the pink test booklet only.




6. A tightrope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point $A$ to point $B$, is illuminated by a spotlight 70 feet above point $A$, as shown in the diagram.

   (a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)

   (b) How far from point $A$ is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)

   (c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee building when she is 10 feet from point $B$? (Indicate units of measure.)

## 1991 - BC1

1. A particle moves on the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 12t^2 - 36t + 15$. At $t = 1$, the particle is at the origin.

   (a) Find the position $x(t)$ of the particle at any time $t \geq 0$.

   (b) Find all values of $t$ for which the particle is at rest.

   (c) Find the maximum velocity of the particle for $0 \leq t \leq 2$.

   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

## 1991 - BC2

2. Let $f$ be the function defined by $f(x) = xe^{1-x}$ for all real numbers $x$.

   (a) Find each interval on which $f$ is increasing.

   (b) Find the range of $f$.

   (c) Find the $x$-coordinate of each point of inflection of the graph of $f$.

   (d) Using the results found in parts (a), (b), and (c), sketch the graph of $f$ in the $xy$-plane provide below. (Indicate all intercepts.)

## 1991 - BC3

3. Let $R$ be the shaded region in the first quadrant enclosed by the $y$-axis and the graphs of $y = \sin x$ and $y = \cos x$, as shown in the figure above.

   (a) Find the area of $R$.

   (b) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

   (c) Find the volume of the solid whose base is $R$ and whose cross sections cut by planes perpendicula to the $x$-axis are squares.

## 1991 – BC 4

4. Let $F(x) = \int_1^{2x} \sqrt{t^2 + t}\, dt$.

(a) Find $F'(x)$.

(b) Find the domain of $F$.

(c) Find $\lim_{x \to \frac{1}{2}} F(x)$.

(d) Find the length of the curve $y = F(x)$ for $1 \le x \le 2$.

## 1991 – BC 5

5. Let $f$ be the function given by $f(t) = \dfrac{4}{1 + t^2}$ and $G$ be the function given by $G(x) = \int_0^x f(t)dt$.

(a) Find the first four nonzero terms and the general term for the power series expansion of $f(t)$ about $t = 0$.

(b) Find the first four nonzero terms and the general term for the power series expansion of $G(x)$ about $x = 0$.

(c) Find the interval of convergence of the power series in part (b). (Your solution must include an analysis that justifies your answer.)

## 1991 – BC 6

6. A certain rumor spreads through a community at the rate $\dfrac{dy}{dt} = 2y(1 - y)$, where $y$ is the proportion of the population that has heard the rumor at time $t$.

(a) What proportion of the population has heard the rumor when it is spreading the fastest?

(b) If at time $t = 0$ ten percent of the people have heard the rumor, find $y$ as a function of $t$.

(c) At what time $t$ is the rumor spreading the fastest?